
Radiation Belt Environment Model

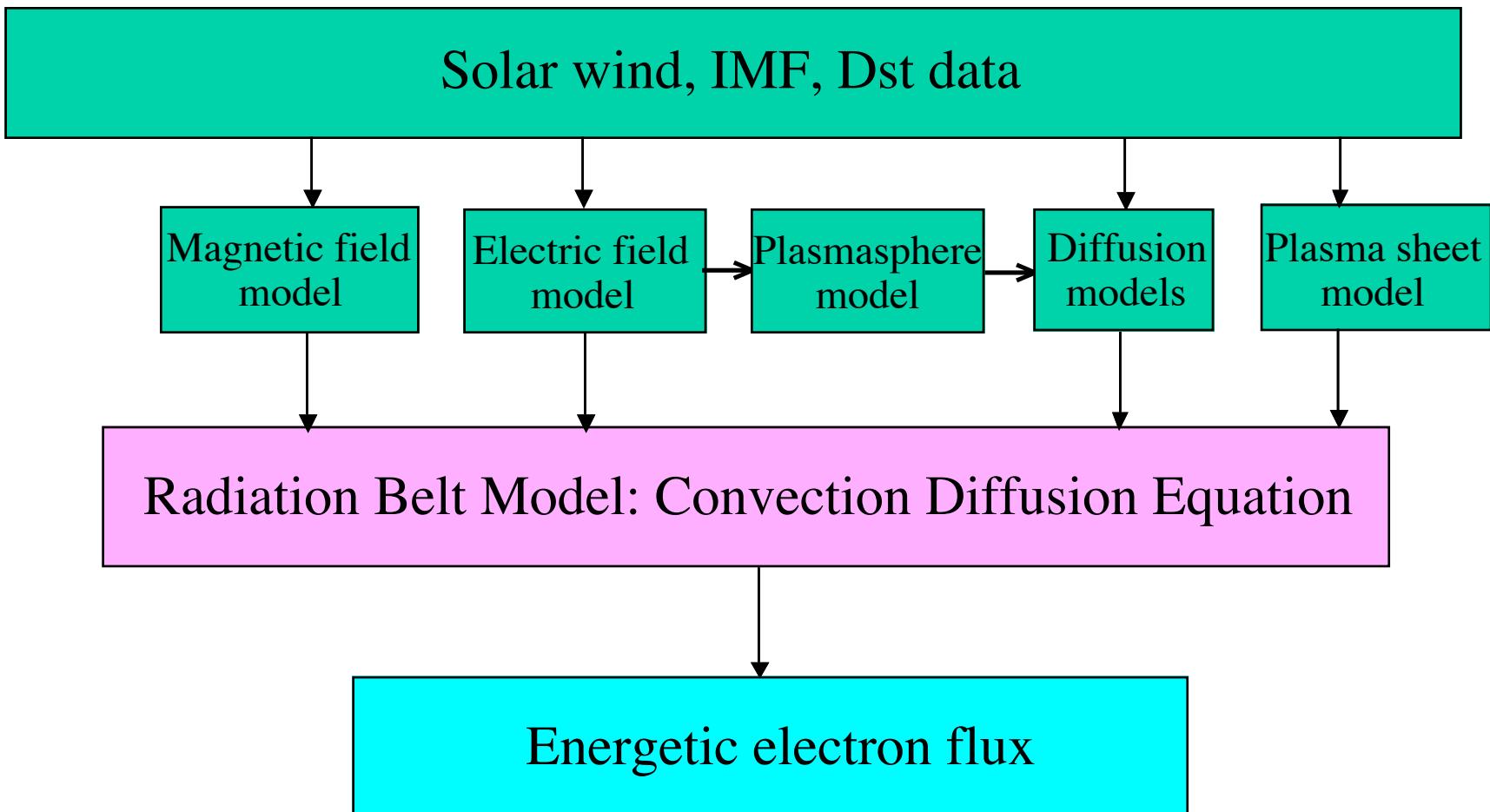
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The Radiation Belt Environment (RBE) Model

- Model equation
- Model Output
- Model-data comparison
- Code structure
- Numerical schemes
- Couple RBE with BATS-RUS-RCM model

RBE Model Logic



RBE Model Equation

$$\frac{\partial f_s}{\partial t} + \left\langle \dot{\lambda}_i \right\rangle \frac{\partial f_s}{\partial \lambda_i} + \left\langle \dot{\phi}_i \right\rangle \frac{\partial f_s}{\partial \phi_i} = \frac{1}{\sqrt{M}} \frac{\partial}{\partial M} \left(\sqrt{M} D_{MM} \frac{\partial f_s}{\partial M} \right) + \frac{1}{T(y) \sin 2\alpha_o} \frac{\partial}{\partial \alpha_o} \left(T(y) \sin 2\alpha_o D_{\alpha_o \alpha_o} \frac{\partial f_s}{\partial \alpha_o} \right) - \left(\frac{f_s}{0.5 \tau_b} \right)_{\text{loss cone}}$$

$f_s = f_s(t, \lambda_i, \phi_i, M, K)$: phase space density of species s

λ_i : magnetic latitude at ionosphere

ϕ_i : magnetic local time at ionosphere

M : magnetic moment

$K = J / \sqrt{8m_o M}$, J is the second adiabatic invariant

α_o : equatorial pitch angle

τ_b : bounce period

Electric and Magnetic Field Models

$$\frac{\partial f_s}{\partial t} + \left\langle \dot{\lambda}_i \right\rangle \frac{\partial f_s}{\partial \lambda_i} + \left\langle \dot{\phi}_i \right\rangle \frac{\partial f_s}{\partial \phi_i} = \frac{1}{\sqrt{M}} \frac{\partial}{\partial M} \left(\sqrt{M} D_{MM} \frac{\partial f_s}{\partial M} \right) + \frac{1}{T(y) \sin 2\alpha_o} \frac{\partial}{\partial \alpha_o} \left(T(y) \sin 2\alpha_o D_{\alpha_o \alpha_o} \frac{\partial f_s}{\partial \alpha_o} \right) - \left(\frac{f_s}{0.5 \tau_b} \right)_{\text{cone}}^{\text{loss}}$$

- Electric Field Model - Weimer 2000 Model
 - convection is updated every time step (3 seconds)
- Magnetic Field Model - T96, T04_S
 - magnetic field (magnetic drift, particle energy) is updated every 5 minutes

Boundary Condition

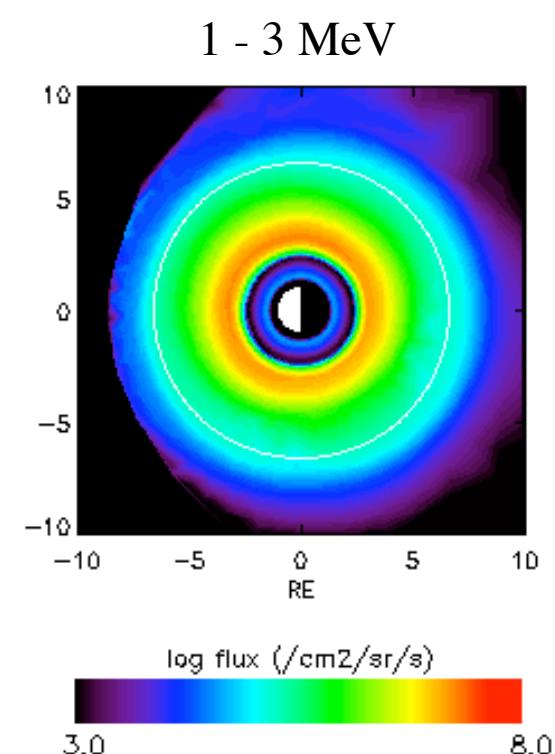
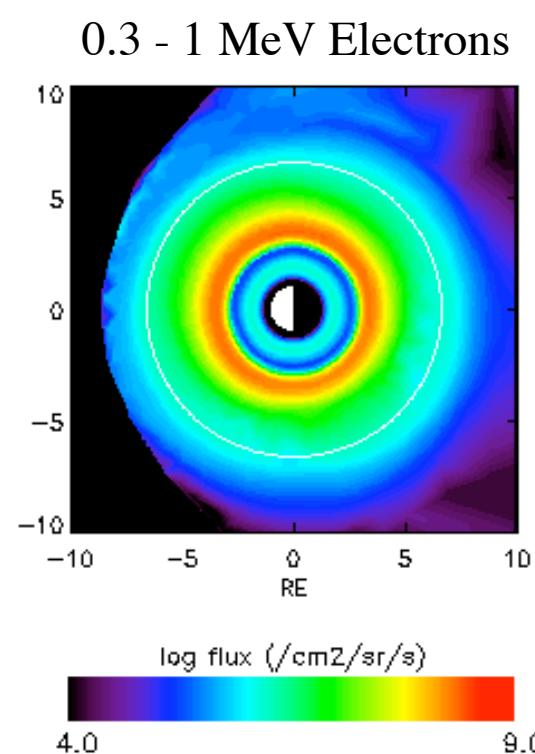
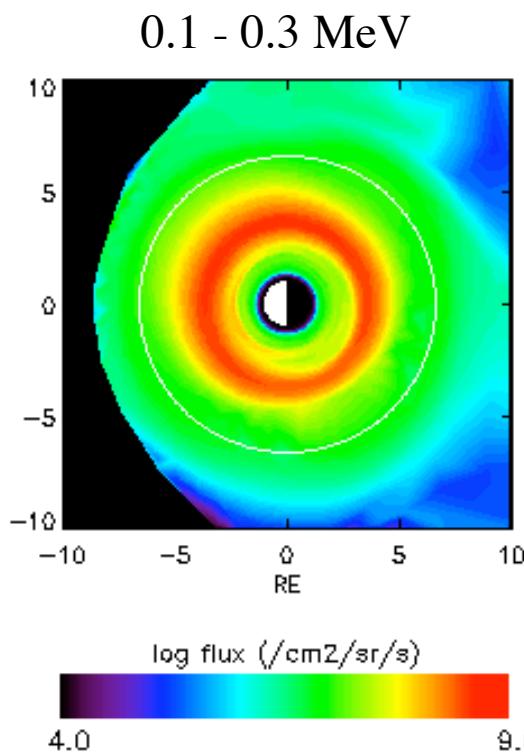
Plasma sheet distribution of nightside boundary is assumed to be a kappa distribution ($\kappa=3$) with density and characteristic energy driven by solar wind:

$$N_{ps}(t) = [0.02*N_{sw}(t-t_d) + 0.316]*\sqrt{amu}$$

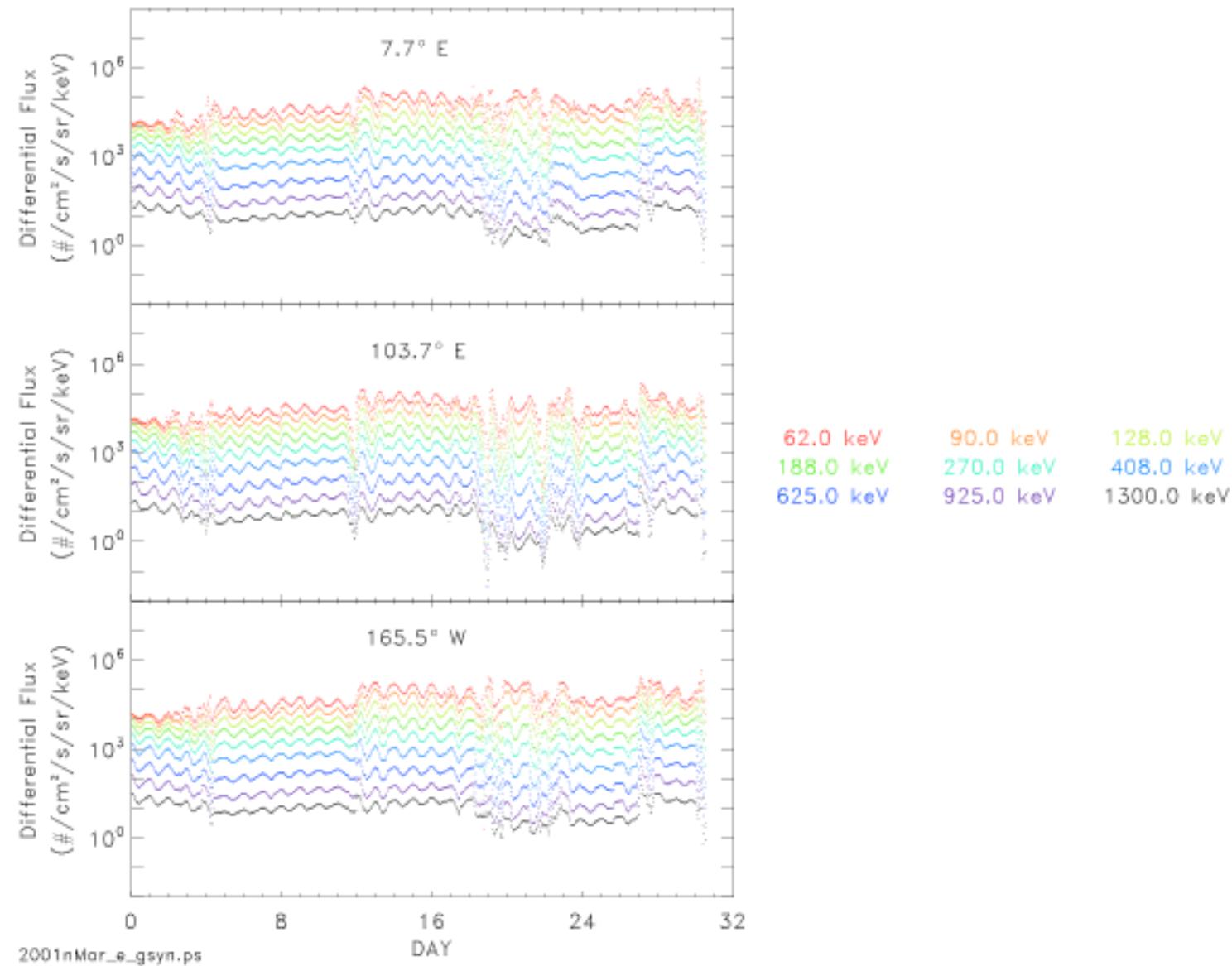
$$E_0(t) = 0.0128*V_{sw}(t-t_d) - 1.92$$

where t_d is the time delay

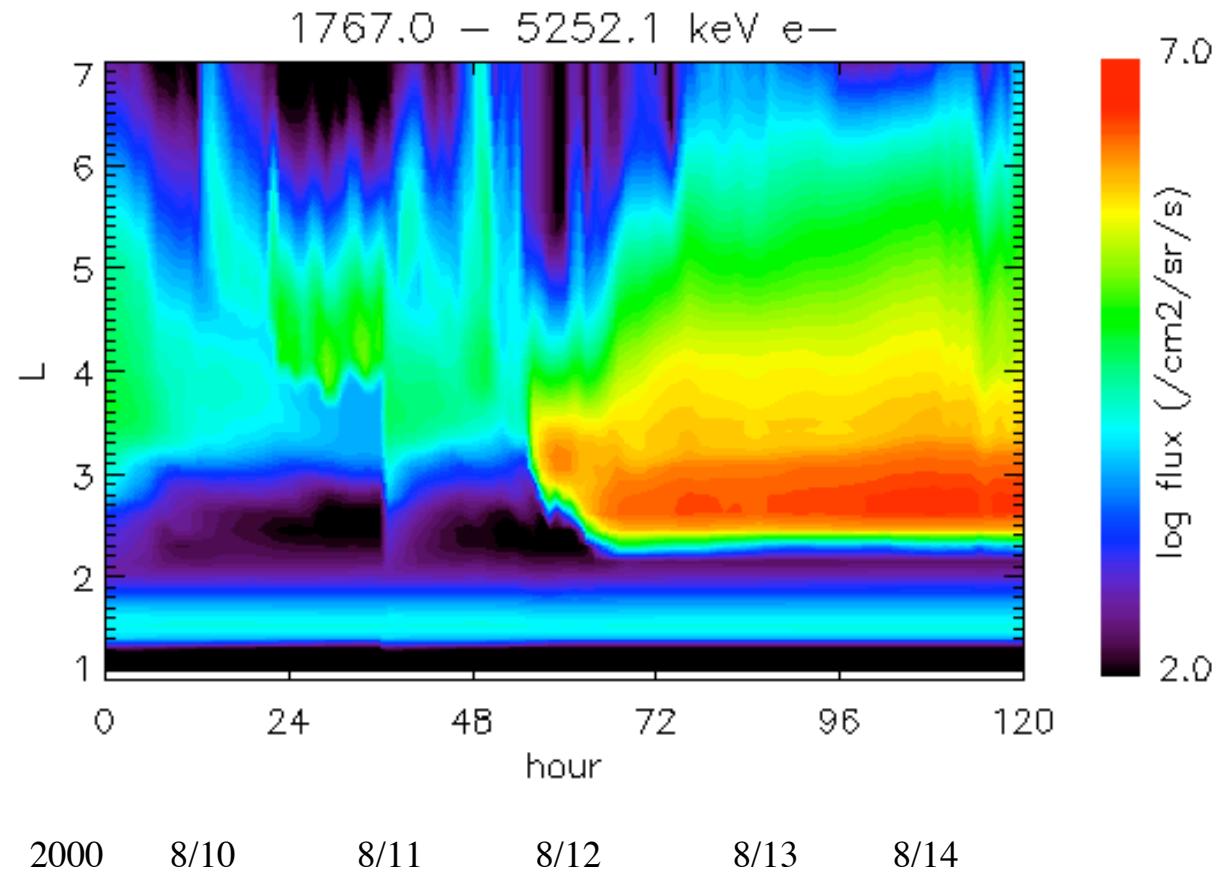
RBE Model Output: Equatorial Flux



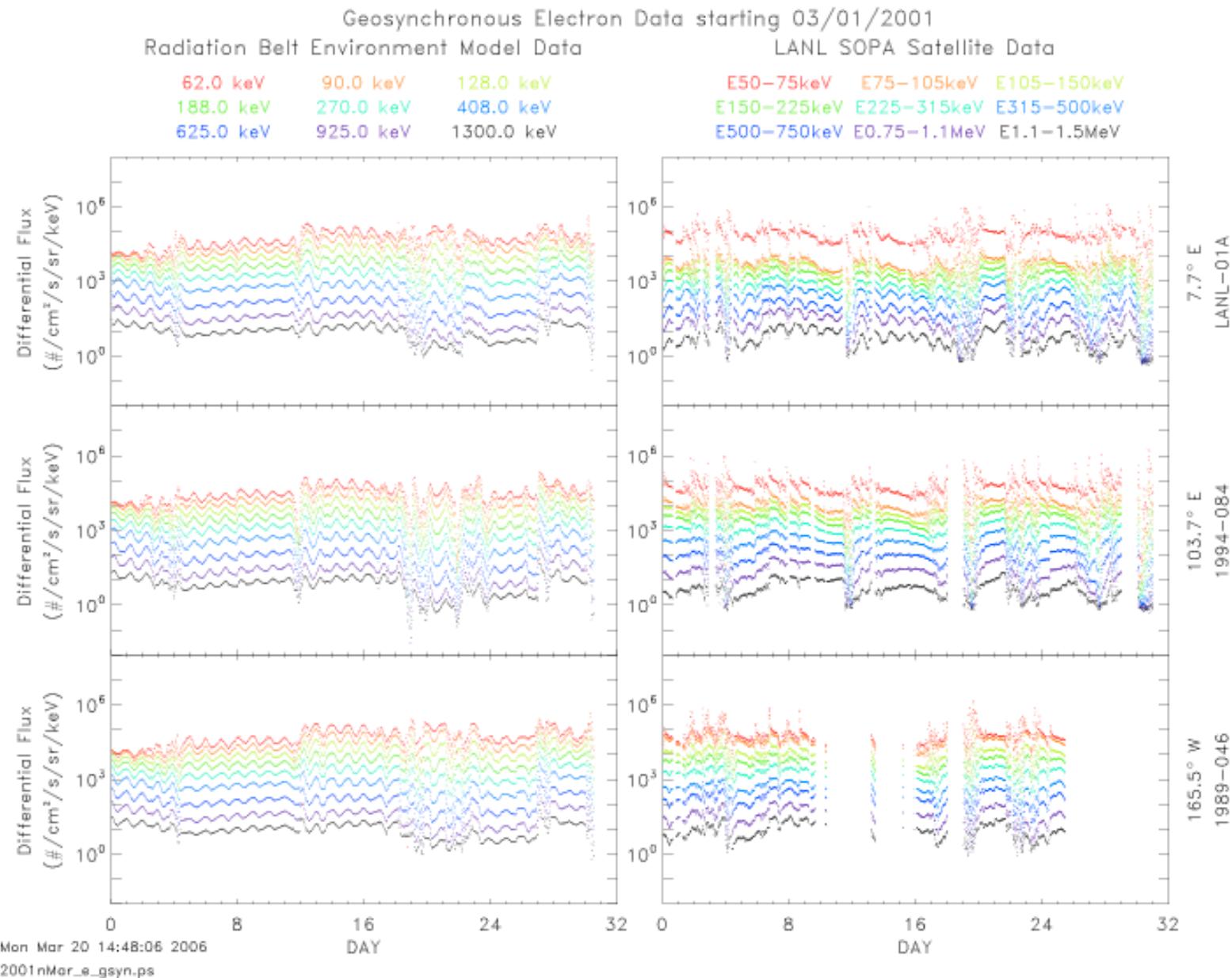
RBE Model Output: Geosynchronous Flux



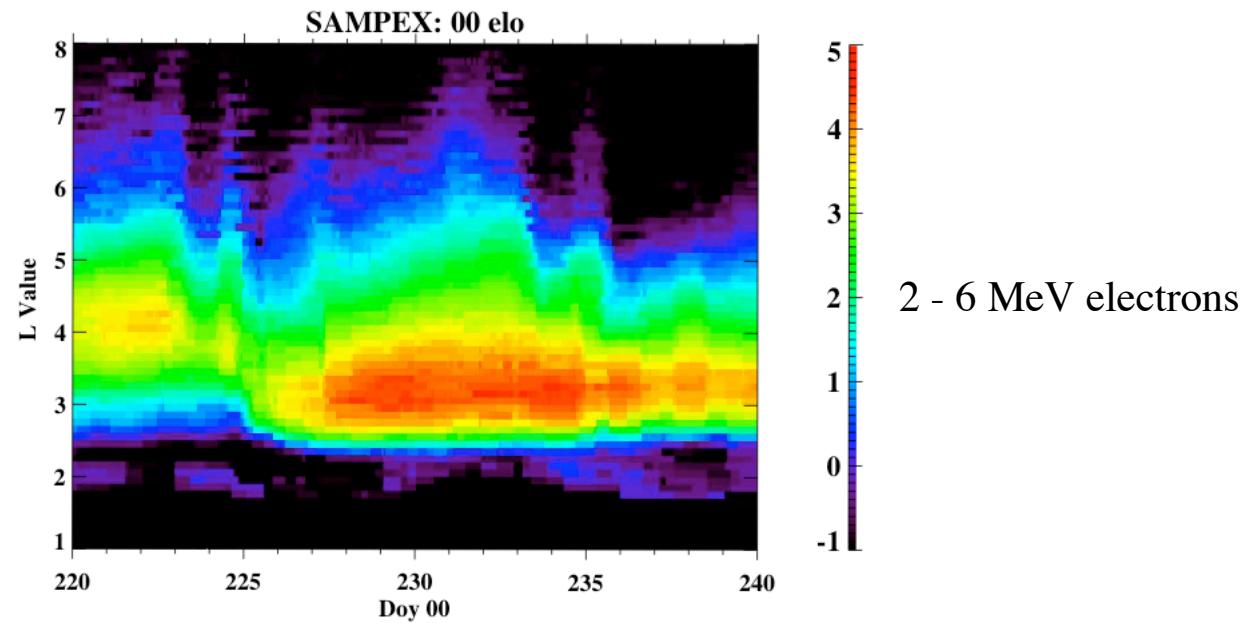
RBE Model Output: L-Time Plot



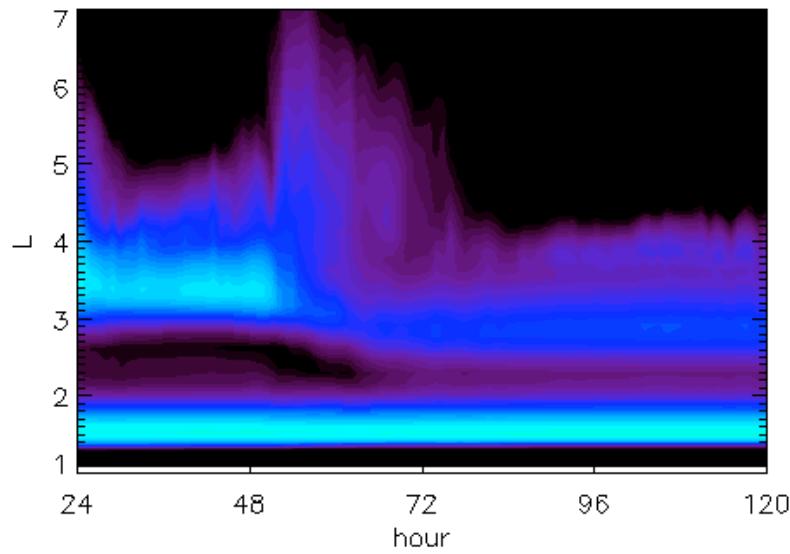
Model-Data Comparison: RBE-LANL SOPA



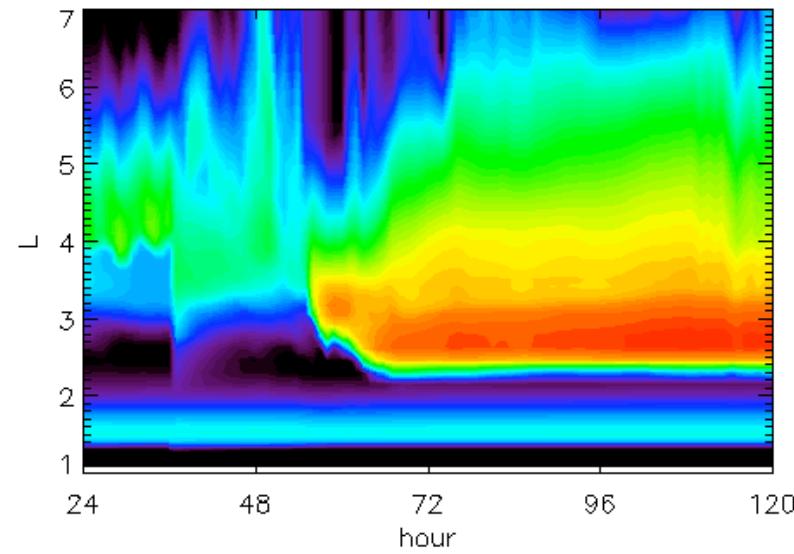
Model-Data Comparison: RBE-SAMPLEX



RBE with T96



RBE with T04S



log flux ($\text{cm}^2/\text{sr}/\text{s}$)

7.0

2.0



RBE Model: Code Structure

$$\frac{\partial f_s}{\partial t} + \left\langle \dot{\lambda}_i \right\rangle \frac{\partial f_s}{\partial \lambda_i} + \left\langle \dot{\phi}_i \right\rangle \frac{\partial f_s}{\partial \phi_i} = \frac{1}{\sqrt{M}} \frac{\partial}{\partial M} \left(\sqrt{M} D_{MM} \frac{\partial f_s}{\partial M} \right) + \frac{1}{T(y) \sin 2\alpha_o} \frac{\partial}{\partial \alpha_o} \left(T(y) \sin 2\alpha_o D_{\alpha_o \alpha_o} \frac{\partial f_s}{\partial \alpha_o} \right) - \left(\frac{f_s}{0.5 \tau_b} \right)_{\text{loss cone}}$$

Time loop

 Update B, E fields, boundary flux

 call drift

 call diffusion

 call losscone

$t = t + dt$

 call losscone

 call diffusion

 call drift

$t = t + dt$

End Time Loop

Numerical Scheme for Advection Equation

$$\frac{\partial f_s}{\partial t} + \left\langle \dot{\lambda}_i \right\rangle \frac{\partial f_s}{\partial \lambda_i} + \left\langle \dot{\phi}_i \right\rangle \frac{\partial f_s}{\partial \phi_i} = 0 \quad \rightarrow \quad \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y} = 0$$

Discrete Approximation :

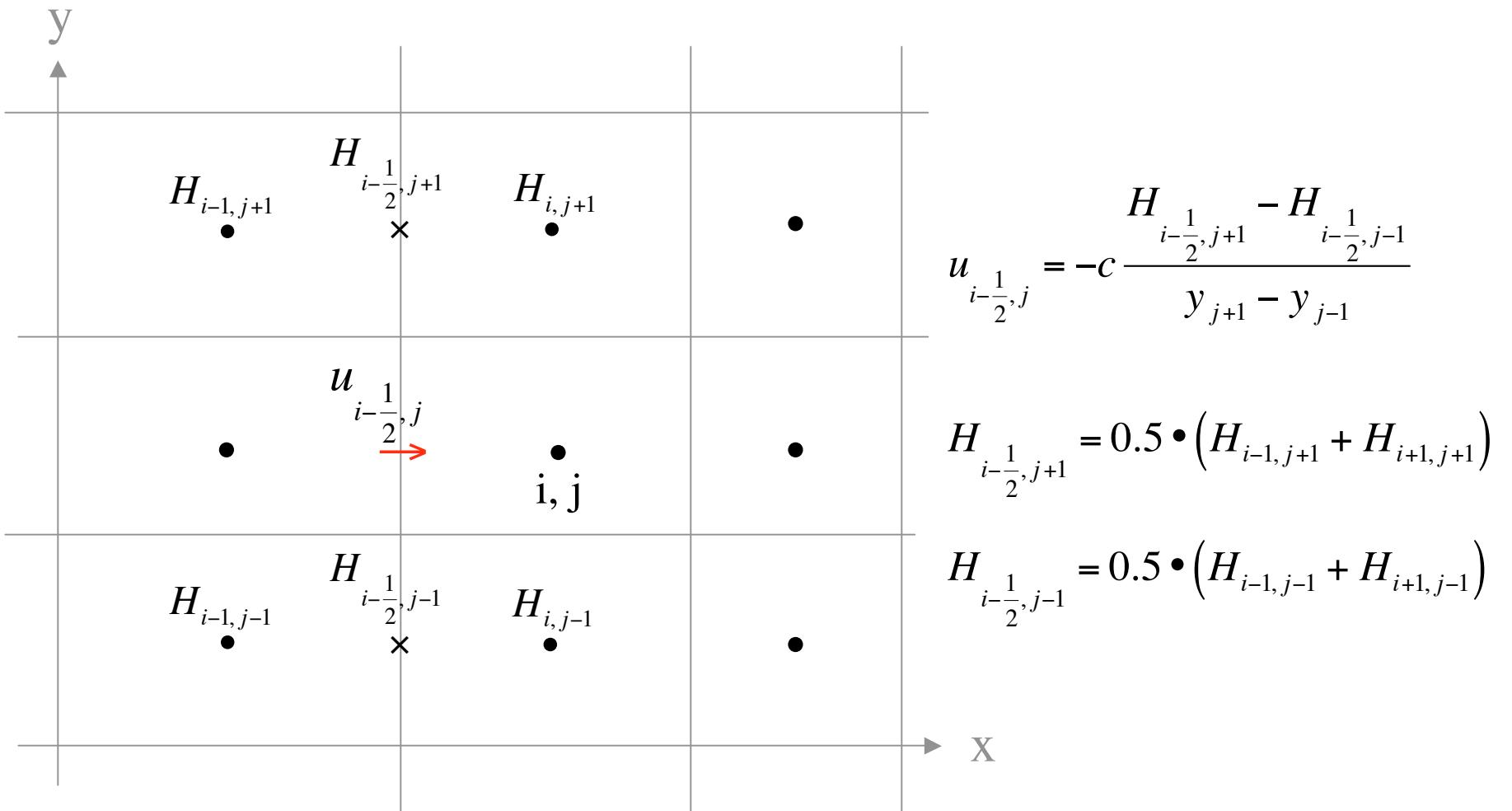
$$f_{i,j}^{n+1} = f_{i,j}^n + \frac{\Delta t}{\Delta x} \left(u_{i-\frac{1}{2},j} f_{i-\frac{1}{2},j}^n - u_{i+\frac{1}{2},j} f_{i+\frac{1}{2},j}^n \right) + \frac{\Delta t}{\Delta y} \left(v_{i,j-\frac{1}{2}} f_{i,j-\frac{1}{2}}^n - v_{i,j+\frac{1}{2}} f_{i,j+\frac{1}{2}}^n \right)$$

The challenge is to find the numerical fluxes at grid boundaries :

i.e., $F_{i-\frac{1}{2},j} = u_{i-\frac{1}{2},j} f_{i-\frac{1}{2},j}^n$

Advection Equation: Velocity at Grid Boundary

$$u = -c \frac{\partial H}{\partial y}, \quad v = c \frac{\partial H}{\partial x}, \quad H = \text{Hamiltonian}$$



Advection Equation: f at Grid Boundary

$$F_{i-\frac{1}{2},j} = u_{i-\frac{1}{2},j} f_{i-\frac{1}{2},j}$$

$$f_{i-\frac{1}{2}} = \left(f_{i-\frac{1}{2}} \right)_{\text{up}} + \psi_{i-\frac{1}{2}} \cdot \left(\left(f_{i-\frac{1}{2}} \right)_{\text{LW}} - \left(f_{i-\frac{1}{2}} \right)_{\text{up}} \right)$$

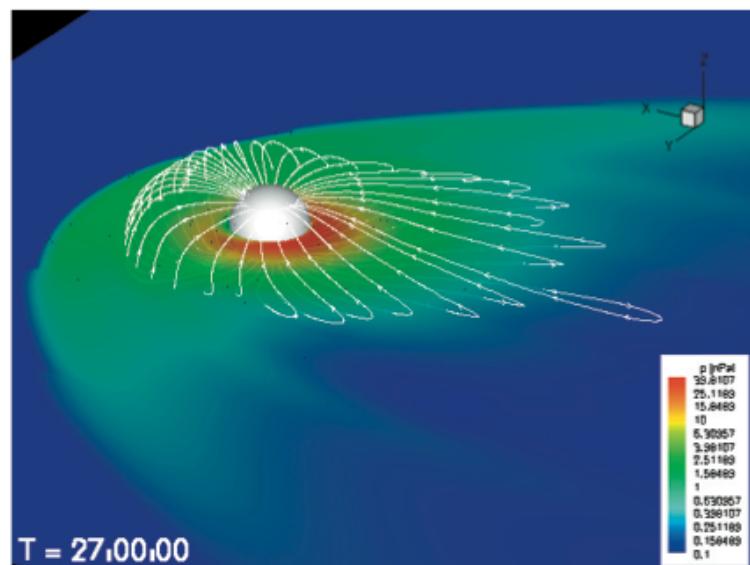
ψ : limiter function

up : first - order upwind scheme

LW : second - order Lax - Wendroff scheme

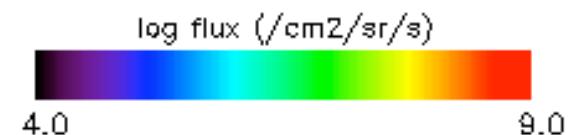
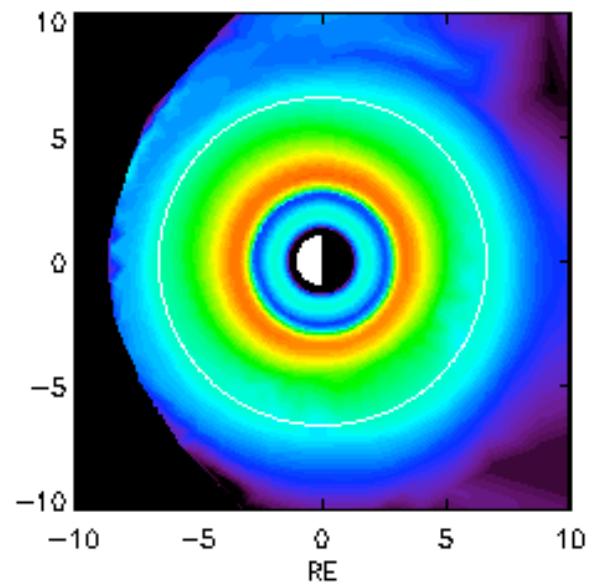
Driving the RBE Model with BATSRUS-RCM

Batsrus-RCM



B, Φ_i
→
 n, T

RBE



0.3 - 1 MeV Electrons

Summary

- The Radiation Belt Environment (RBE) model is a kinetic model that solves the convection-diffusion equation of energetic (10 keV - 4 MeV) electrons.
- The RBE model is able to reproduce observed radiation belt variations during quiet and active periods.
- The performance of the RBE model will be improved when wave-particle interactions are included.
- **The RBE model can easily be integrated into the BATS-RUS-RCM and serves as a key module in the Space Weather Modeling Framework.**